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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2018

SECOND YEAR [BATCH 2017-20]

Date : 22/12/2018 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

[Use a separate Answer Book <u>for each Group</u>]

<u>Group – A</u>

(*Answer <u>any four</u> questions*) [4×5]

1. Perpendiculars *PL*, *PM*, *PN* are drawn from the point *P*(*a*, *b*, *c*) on the coordinate planes. Show that the equation of the plane *LMN* is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$. [5]

2. Find the equation of the plane whose *x*-intercept is 3 unit and which passes through the points $(2,-1,3)$ and $(3,-2,5)$. [5]

3. Find the image of the point $(-3,5,2)$ in the plane $2x-y+z=0$. [5]

4. Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y+1}{4}=\frac{z+4}{5}$ and $\frac{x-7}{2}=\frac{y-4}{3}=\frac{z-3}{4}$. [5]

5. Find the greatest and the least distances of the point $(2,-1,3)$ from the sphere $x^2+y^2+z^2-2x-2y-4z+2=0$. [3+2]

6. Find the equation of the right circular cone whose vertex is the origin, axis is the *y*-axis and semi vertical angle is 60^o. [5]

<u>Group – B</u>

(*Answer <u>any four</u> questions*) [4×10]

7. a) Food *X* contains 6 units of vitamin *A* and 7 units of vitamin *B* per gram and costs Rs. 12/gm. Food *Y* contains 8 units and 15 units of *A* and *B* per gram respectively and costs Rs. 22/gm. The daily requirements of vitamin *A* and *B* are at least 80 units and 100 units respectively. Formulate the above as an L.P.P. to minimize the cost. [5]

   b) Put the following problem in a standard maximization form: [5]

$$\text{Minimize } z=3x_1-4x_2-x_3$$
$$\text{Subject to } x_1+3x_2-4x_3\le 12$$
$$x_1+x_2-2x_3\le 20$$
$$x_1-4x_2-5x_3\ge 5$$
$$x_1\ge 0, x_2, x_3 \text{ are unrestricted in sign.}$$

8. a) Solve the following L.P.P. graphically:

$$\text{Maximize } z=3x_1-x_2$$
$$\text{Subject to } 2x_1+x_2\ge 2$$
$$x_1+3x_2\le 2$$
$$x_2\le 4$$
$$x_1, x_2\ge 0$$

[6]

b) Find all the basic solutions of the following equations:

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 5x_2 - 2x_3 = 3$$

Which of them are feasible if $x_1, x_2, x_3 \geq 0$? [3+1]

9. a) Prove that the set of all feasible solutions of a linear programming problem is a convex set. [3]

b) Use Charnes Big-M method to solve the following L.P.P. [7]

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

10. a) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ of the system of equations: [6]

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8$$

to two basic feasible solutions.

b) Prove that in $\mathbb{R}^2$, the set given by $X = \{(x_1, x_2) \mid 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. [4]

11. a) Prove that the dual of the dual is the primal. [4]

b) Obtain the optimal basic feasible solution of the following transportation problem (Use VAM to find initial BFS) [6]

	D_1	D_2	D_3	D_4	
O_1	23	27	16	18	30
O_2	12	17	20	51	40
O_3	22	28	12	32	53
	22	35	25	41	

12. a) Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix: [5]

	M_1	M_2	M_3	M_4
J_1	10	24	30	15
J_2	16	22	28	12
J_3	12	20	32	10
J_4	9	26	34	16

b) Find the dual of the following primal problem: [5]

Minimize $z = 4x_1 + 5x_2 - 3x_3$

Subject to : $x_1 + x_2 + x_3 = 22$

$$3x_1 + 5x_2 - 2x_3 \leq 65$$
$$x_1 + 7x_2 + 4x_3 \geq 120$$
$$x_1, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted}$$

13. a) Obtain initial basic feasible solution of the following transportation problem by matrix minima method: [4]

	A	B	C	a_j
O_1	10	9	8	8
O_2	10	7	10	7
O_3	11	9	7	9
O_4	12	14	10	4
b_j	10	10	8	

b) Using simplex method, show that the following L.P.P. has unbounded solution: [6]

Maximize $z = 2x_1 + 3x_2 + x_3$

subject to $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

$x_1, x_2, x_3 \geq 0$

Group – C

(*Answer any three questions*) [3×5]

14. a) Round off the following numbers to three significant figures: [3]
0.01201, –239.85, 0.004923

b) Taking $\pi = 3.14$ instead of 3.14156, find the absolute error and the relative error. [2]

15. Values of x (in degrees) and Sin x are given in the following table:

x (in degrees)	Sin x
15	0.2588190
20	0.3420201
25	0.4226183
30	0.5
35	0.5735764
40	0.6427876

Determine the value of Sin 38^o by taking suitable interpolation polynomial formula. [5]

16. Find the Lagrangian interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following table of values. Hence determine the value of ln 2.7. [5]

x	$y = \ln x$
2	0.69315
2.5	0.91629
3.0	1.09861

17. Using Trapezoidal rule, calculate the value of the integral $\int_0^1 \frac{dx}{1+x}$ taking four sub-intervals, correct upto five decimal places. [5]

18. Compute by Newton-Raphson method a positive root of the equation $x^3 - 9x + 1 = 0$, correct upto three significant figures. [5]

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